Stochastic Models (IA/0134/EN)

Sample of candidate Theoretical Questions

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A.A 2021/2022

Abstract

The aim of this document is to help the student during the exam's preparation by giving to him/her an idea on which theoretical concepts, and aspects of the course s/he should pay particular attention. The document is organized as follows: In Section 1 is shortly recalled the Exam's organization. Then in Section 2 is reminded the importance of the Theory in understanding concepts. Then, in the last Section 3, is provided a list some potential theory questions per module

1 Exam's Organization

The final examination of the Stochastic Models course consists of an oral discussion. The exam begins by asking the candidate to talk (in English) about a topic of his/her choice, among the main ones of the course. The topic is free and can touch also more than one modules seen during the course. In general this part takes 20/25 minutes. No slides, it must be an informal discussion where you have to convince me you have taken consciousness of concepts you are presenting.

Then, on the basis of the quality of the presentation, the exam continues by asking the candidate to sketch out the solution of some exercises. Exercises are similar to those seen in class. Moreover also related questions on theory can be done.

Before giving a list of potential questions of Theory for the exam, in Section 2 of this document, is given a reminder to Students on why understading the "Theory" in fundamental for understanding concepts.

2 What is Theory?

One of the major purposes of "Theory", independently from the particular field of study we are dealing with, is to provide an answer to the question "Why?".

Asking, "Why?", to increase your knowledge of a subject area allows to realign your thoughts and opinions on a topic, and it is an essential skill for anybody who wants to learn and develop.

It is also worth to remark that developing theories is the human being's response to their innate nature to ask themself "Why?", since the childhood.

- *Can you get ready for bed now?*

- *Why is snow cold?*
- *Why is the sky blue?*

Although questions like these, can be endless, behind them, it is evident the child's first attempt to understand the world.

Although there are no hard and linear rules, modern theories are usually developed through a series of steps, and it generally starts from the "observation" of a given phenomenon, which is in general the effect, not the cause we are looking for, and it passes through several phases, like the "description of the phenomenon", the development of "hypothesis", the modification of these hypothesis, till the acceptation of the Theory by a wider community of people. Moreover, it is also worth to remark that, to support all these phases of the process, the use of a common (and international) language is cleary needed. That languange is the "math", which by means of its formalism, rules, operators, allows to formulate problems, states Theorems and characterize the validity ranges of the results developped.

After this "pippone" (mind masturbation) born and raised in the folds of my brain, let us now list some potential question of theory, per topic of the course, which will help you during your preparation.

Let us further note that the ability to provide an answer the following questions is functional to properly approach every numerical assignment.

3 List of potential theory question per topic

3.1 Module of "Probability theory"

- Describe the meaning of experiment, trial, outcome, sample space and event, and that of probability function.
- Provide an interpretation of the following probability rules and laws: Addition's Rule, Conditional probability law, Law of total probability, Bayes' theorem.
- Explain the difference between independent and mutually exclusive events
- On the basis of their formal definition, provide an interpretation for the concept of "Mean" and that of "Variance".
- Provided the definition of random variable in either the discrete or continuous case, then describe the differences
- Provides an example and/or a physical interpretation for the selected probability distributions seen in the course (formulas of the event probability, mean and variance will be provided by the examiner if necessary)

3.2 Module of "Link Dimensioning"

- Provided an interpretation and explain the differences between the Markov an the Chebyshev inequalities
- Provide an interpretation for the Central limit theorem. Then explain the difference to the Lyapunov's central limit theorem (no formulas are needed)
- Discuss the project of Link Dimensioning in the case of homogeneous individual sources. Then, generalize the treatment for non-homogeneous sources

3.3 Module of "Stochastic processes"

- Provide the definition of stochastic process, then discuss its meaning
- Discuss the importance of the Joint probability function in the context of stochastic processes
- Explain the differences between a strictly stationary process and, a simply stationary process.
- Discuss the reasons why the ergodicity property is often desired in practice
- Provide the definition of Markovian process than discuss its properties by means of some examples
- Discuss the properties a Poisson process, then show its connection with the Exponential distribution
- Discuss the differences between a random and regular splitting of a Poisson process

3.4 Module of "Discrete-time Markov chains"

- Provides the definition of discrete-time Markov chain, then by means of a simple example discuss its key properties
- Discuss the differences between a transient, a recurrent periodic, and a recurrent aperiodic state of a DT-MC
- Discuss the differences between a reducible and a irreducible DT-MC
- Discuss the differences between an ergodic DT-MC, and an irreducible periodic DT-MC and an irreducible aperiodic DT-MC
- Provide the definitions and then discuss the differences between the stationary and the limiting distribution of a DT-MC. Then, under which condition a limiting distribution is unique.
- Discuss how to evaluates the ergodicity in a DT-MC
- Discuss the main characteristics of a discrete-time birth-death processes
- Provide an interpretation for the mean hitting time and for the mean recurrence time a state in a DT-MC

3.5 Module of "Continuous-time Markov chains"

- Provides the definition of CT Markov chain, then by means of some example discuss its key properties
- Discuss the connections between the transition rate matrix and the transition probabilities of a CT-MC. Then, under the hypothesis that the process is time-homogenous, discuss how the system analysis becomes easier.
- Provide the definitions and then discuss the differences between the stationary and the limiting distribution of a CT-MC. Then, under which condition a limiting distribution is unique
- Discuss how to evaluates the ergodicity in a CT-MC
- Discuss the main characteristics of a time-homogenous continuous-time birth-death processes and its connection with a M/M/1 resource
- Model a M/M/3 queue as a CT-BDP
- Discuss the ergodicity of a CT-BDP process than derive its limiting distribution
- Provide an interpretation for the mean hitting time and for the mean recurrence time a state in a CT-MC

3.6 Module of "Queueing theory"

- Discuss the functioning of a queue and its characterization by means of the Kendall Notation
- For each quantity of interest of a queue system (see table) provide an interpretation
- Provide the definition and an interpretation of the Little's Law
- Discuss the functioning of a deterministic queue and the conditions such that it behaves as an ergodic process
- Discuss the concept and provide an interpretation of traffic intensity for a M/M/1 and a M/M/∞
- Let $\{X_t, t \geqslant 0\}$ be the random number of arrival of a Poisson Process with mean rate $\lambda > 0$, after a time t, and where

$$\Pr(X_t = k) = \frac{(\lambda t)^k}{k!} \mathrm{e}^{-\lambda t},$$

  prove that the random variable Δt associated with inter-event time is Exponentially distributed with the same rate. Then, provide an interpretation why $\mathsf{E}[\Delta t] = 1/\lambda$.
- Provide an interpretation for the PASTA property
- Discuss the advantages in terms of ρ, $\bar{x}$ abd $th\bar{e}ta$ of the stochastic multiplexing access method with respect to the TDMA.
- Discuss the ergodicity for a M/M/m/m queue
- Provide the definition and an interpretation for blocking probability and abandonement rate in a markovina queue.

3.7 Module of "Queueing networks"

- Discuss the differences between open and closed and mixed queueing networks
- Which is the meaing of the routing probability matrix?
- In the context of a queuing network, provide an interpretation to the following Joint probability

$$\Pr(x_1, x_2, \ldots, x_v) = \Pr(X_1 = x_1, X_2 = x_2, \ldots, X_v = x_n)$$

- Discuss ergodicity conditions for an open and a closed queuing network
- Provide the modellization of a tandem queuing network consisting of two M/M/1 nodes as a CT-MC
- Model a cyclic queuing network consisting of two M/M/3 nodes as a CT-MC, and a population of $n = 5$ costumers
- Why is the main result of Gordon-Newell Theorem?

3.8 Module of "Hidden Markov Chain"

- Provide the definition, then discuss the conceptual differences between an observable Markov Model and Hidden Markov Chain
- Discuss, in words, the conceptual differences between the Viterbi algorithm and the Maximum Likelihood Criteria
- Which are the main difficulties of solving a non-convex optimization problem?
- Which is the idea behind gradient descent solvers for optimization problems?
- Can the BAUM-WELCH Algorithm be used to improve the model of an observable DT-MC?