Stochastic Models

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Combiantion VS permutation

The difference between combinations and permutations is ordering. Both *permutation* and *combination* can be considered *with*, and *without* repetition. We refer to a combination whenever the order doesn't matter. For the sake of clarity let us discuss the next example.

Example: Suppose we are interested to extract $n = 2$ elements at time from the set $S = \{1, 2, 3, 4\}$ which cardinality is $v = \text{card}\{S\} = |S| = 4$.

The set of **permutations with repetitions** (or **simple permutations**) is

$$P_{rep} = \left\{ \begin{array}{cccccccc} (1,1), & (1,2), & (1,3), & (1,4), & (2,1), & (2,2), & (2,3), & (2,4), \\ (3,1), & (3,2), & (3,3), & (3,4), & (4,1), & (4,2), & (4,3), & (4,4) \end{array} \right\}$$

The cardinality of P_{rep} can be computed by the next formula

$$|P_{rep}| = n^v = 2^4 = 16$$

On the other hand, the set of **Permutations without repetitions** do not consider repetitions, namely it neglects the possibility to take the same element more the once. Thus it follows that

$$P_{no\ rep} = \left\{ \begin{array}{cccccc} (1,2), & (1,3), & (1,4), & (2,1), & (2,3), & (2,4), \\ (3,1), & (3,2), & (3,4), & (4,1), & (4,2), & (4,3) \end{array} \right\} \subset P_{rep} \tag{1}$$

The cardinality of $P_{no\ rep}$ is given the formula

$$|P_{rep}| = \frac{v!}{(v-n)!} = \frac{4!}{(4-2)!} = \frac{4 \cdot 3 \cdot 2!}{2!} = 12$$

Let us now consider the set of **combinations with repetition**, that is

$$C_{rep} = \left\{ \begin{array}{ccccc} (1,1), & (1,2), & (1,3), & (1,4), & (2,2), \\ (2,3), & (2,4), & (3,3), & (3,4), & (4,4) \end{array} \right\} \tag{2}$$

Notice that, although repetitions are allowed, since the extraction order doesn't matter then $C_{rep} \subset P_{rep}$. Let us further note that the cardinality of C_{rep} is given by the next binomial coefficient

$$|C_{rep}| = \binom{n+v-1}{n} = \binom{2+4-1}{2} = \binom{5}{2} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3!}{2!3!} = 10$$

Finally set of **combinations without repetitions** is

$$C_{no\ rep} = \{ \ (1,2), \quad (1,3), \quad (1,4), \quad (2,3), \quad (2,4), \quad (3,4) \ \} \tag{3}$$

and its cardinality correspond by the well-known binomial coefficient

$$|C_{no\ rep}| = \binom{v}{n} = \binom{4}{2} = \binom{4}{2} = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2!}{2!2!} = 6$$