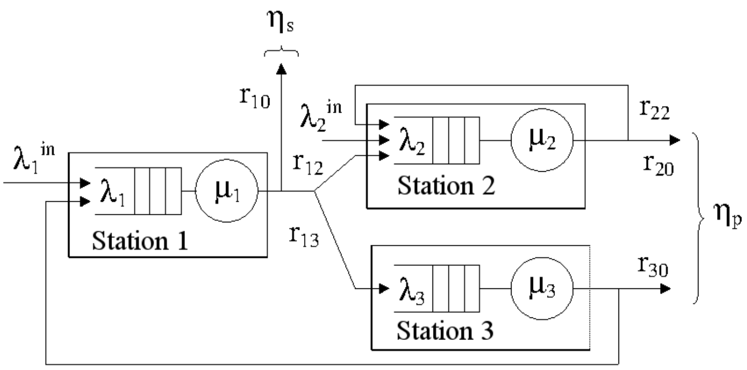
Stochastic Models

A.A. 2019/2020

Assignment 7

Exercise 1. A flexible manufacturing system is modelled by a markovian queueing network consisting of three stations which topology is depicted in the figure below. The outgoing rate from station 1 η_d represents the rate of items rejected because defective. The sum of the throughput (or productivity) of stations 2 and 3, and referred as η_p in the figure, corresponds to the rate of items that leave the system because correctly processed. Stations consist of M/M/1 queues. The system is described by the following parameters: $\lambda_1^{in} = 2\ \mathrm{s}^{-1}$, $\lambda_2^{in} = 1\ \mathrm{s}^{-1}$, $\mu_1 = 3\ \mathrm{s}^{-1}$, $\mu_2 = 4\mathrm{s}^{-1}$, $\mu_3 = 2\ \mathrm{s}^{-1}$, $r_{12} = 0.4$, $r_{13} = 0.5$, $r_{22} = 0.2$, $r_{31} = 0.2$.



(a) Determine the outgoing routing probabilities r_{10}, r_{20} and r_{30}.

(b) Determine the effective arrival rate of items λ_i to each node, with $i = 1, 2, 3$.

(c) Determine traffic intensity ρ_i of each station, then discuss the ergodicity of this network

(d) Determines the mean number of items x_i in each station, and that in the whole system at the steady state.

(e) Determine the mean time spent by an item to cross the network at steady state.

(f) Determine the mean time to cross node 2 from outside at steady state.

(g) Determine the rate of items correctly processed and the rate of production scraps η_s at steady state. Compare the total outgoing rate of items with the total arrival rate from outside the network.

(h) Determine the probability that both station 1 and station 2 are busy at steady state.

(i) Determine the probability that only one pieces is within the network at steady state.

(j) Suppose the system engineer have to replace one of the three servers with a faster server with rate is $\mu = 5\ \mathrm{s}^{-1}$. Determine which is the server that is better to replace in order to:

1. reduce the average number of pieces in network at steady state;
2. reduce the mean time needed to cross the network;
3. reduce the mean time needed to pass through node 2 from outside at steady state;
4. reduce the rate of rejected items η_s;
5. increase productivity η_p.

Exercise 2. A company operates an IT system consisting of three web domains. On average each domain is visited from outside with rates, respectively, $\lambda_1^{in} = 5$ users/s, $\lambda_2^{in} = 2$ users/s, whereas the third domain cannot be accessed from the outside, namely, $\lambda_3^{in} = 0$.

A user connected to the first domain, after carrying out a transaction (namely, after being served) ends the connection with probability 0.8, while with probability 0.2 it is redirected to the second domain. Then, a user connected to second domain, after having done a transaction ends the connection with probability 0.5, otherwise it is redirected to the third domain. Finally, a user connected to the third domain, after carrying out a transaction ends the connection with probability 0.9. Alternatively it is redirected to the first domain. Each domain has a sufficiently large buffer were users may wait before concluding their transactions.

Assume that 3 different servers are available: Server A is able to service $\mu_A = 2.5$ transactions/s. Server B is able to service $\mu_B = 8$ transactions/s. The server is able to service $\mu_C = 6$ transactions/s.

(a) By denoting the service rates of the three web domains as μ_1, μ_2 and μ_3, model the system as a markovian queueing network. Under what hypotheses, so far not mentioned, is this system markovian? Then, classify each resource.

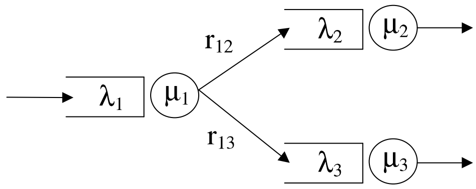
(b) Discuss the ergodicity property of the network in terms of the three available servers with rate μ_1, μ_2 and μ_3. Then identify the possible servers allocations such that the system behaves as ergodic.

(c) Allocate the three servers in order to minimize the mean connection time (that is, the mean time to cross the network) at steady state.

(d) Calculate the idle probability of the system at steady state. Then, evaluate the probability that only one user is in the system.

(e) The third web domain contains copyrighted images, each with a cost of 0.45€. The user transactions consist to pay a desired image, then downloads it. How much does the company earn from this service in an hour?

Exercise 3. The figure below shows an open queueing network consisting of three M/M/1 nodes denoted by $i = 1, 2, 3$, such that $\mu_1 = 5$, $\mu_2 = 4$, $\mu_3 = 1$, $\lambda_1 = 4$.



Consider the following three use cases:

(a) Assume the routing probabilities be as follows $r_{12} = 0.9$, $r_{13} = 0.1$.

(b) Assume that r_{12} and r_{13} are selected to minimize the mean number of costumers in the network at steady state.

(c) Assume a "smart" routing discipline able to forward each departure from node 1 to either node 2 or 3 on the basis of which of the two has the shortest queue. In this case the parallel between node 2 and 3 can be correctly approximated by a M/M/2 queue which mean service time is $2/(\mu_2 + \mu_3)$.

For each use case, determine: the mean number of costumers in each resource, the utilization factor of each resource and the mean time spent by a costumer to cross the network.

Exercise 4. Consider a closed queueing network consisting of three identical M/M/1 nodes and which service rates are denoted by $\mu_i = \mu$. Let $n = 2$ be the size of the costumer's population. The interconnections between the two stations are characterized by the following routing probability matrix:

$$\boldsymbol{R} = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.25 & \alpha & 0.75 - \alpha \\ 0.25 & 0.75 - \beta & \beta \end{pmatrix} \qquad \alpha, \beta \in [0,\ 0.75].$$

(a) Determines the ergodicity conditions for this network.

(b) Provides a Continuous-Time Markov Chain (CT-MC) modelization of the system.

(c) Then, in the case that $\alpha = \beta = 0.75$, determines its steady-state probabilities.