Stochastic Models

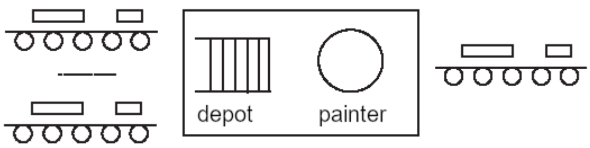
A.A. 2019/2020

Assignment 6

Exercise 1. Consider a wireless system that provides channels each of which can serve one phone call. There are 100 users making phone calls. Each user makes on average one phone call per hour and the average duration of a phone call is 3 minutes. What is the total traffic in Erlang that the 100 users generate.

Exercise 2. A painting system of slabs serves 2 production lines as in the figure. After the service, the departure process consist of a single flows of painted slabs. Assuming that:

1. arrivals from each line, at steady-state, becomes Poisson with mean rate 9slabs/hour;
2. the slabs' surface is exponentially distributed with average of 15cm^2;
3. the painting process needs on average 12 seconds per cm^2;
4. if the server is busy, slabs are stored in a waiting line of sufficient capacity.



Determines:

(a) the transition graph of the resulting birth-death process that describes the system;

(b) the arrival rate (assuming it may vary) for which the system would no longer be ergodic;

(c) the mean time spent by an slab in the system at steady state;

(d) the mean number of slabs in the depot at steady state;

(e) the number of slabs in the depot that, at steady state, is not exceed for the 68%, the 90% and 99% percent of the time, with respect to the time unit of reference.

Exercise 3. Under the hypothesis of Poisson distributed arrivals with mean rate $\lambda = 1$ task per second, exponential service durations, and infinite buffer capacity, determine the mean service rate μ of an informatic system so that the mean time to accomplish that task, on average, does not exceed, 0.5 seconds.

Exercise 4. In a point-to-point communication system data are send with constant speed $c = 1200$ bits/sec. A data packet has L bits. The packet length L is an exponential random variable with mean $\mathsf{E}[L] = 600$ bits.

Assuming that the packets arrive as generated by a Poisson process, and that the data may be buffered in stack with infinite capacity, determine the maximum data rate for the incoming packets λ to guarantee a waiting time in the buffer no greater than 1 second.

Exercise 5. A productive process consisting of a single server machine and a waiting line with limited capacity have to be designed. The engineer may choose among the three machines:

- M_1: $\mu = 0.5$ piece/min, cost 100 K€;
- M_2: $\mu = 1.2$ piece/min, cost 300 K€;
- M_3: $\mu = 2.0$ piece/min, cost 500 K€.

The a single unit space in the waiting area costs 80 K€. The project specification requires that, at steady state, no more than the 10% of the incoming pieces may be diverted to another service station due to stack over-flow.

The pieces arrive in a Poisson fashion with parameter $\lambda = 1$ pieces per minute. The service time is independent form the arrivals and follow an exponential distribution with average rate μ. Determine which machine and the size of the waiting line that minimize the costs while satisfying the project specifications.

Exercise 6. About 80 customers per hour arrive to an ice-cream shop the weekend afternoons.

The manager may hire a very efficient waiter able to serve a customer in 30 seconds, or, at the same cost, two waiters each able to serve on average a customer in 1 minute. Customers, in both cases, would be placed in a single row.

(a) Determines, for the two cases, the average time waste by a customer to buy an ice cream. Arrivals and services are assumed to be independent and Poissonian;

(b) Which of the two solutions would be preferred in terms of quality of the service offered?

(c) For the two cases evaluate: the resource utilization, the single server utilization and the idle rate and the probability that the service area is busy. Then compare the results and provide an interpretation.

Exercise 7. In a point-to-point digital communication system packets arrives to a repeated node following a Poisson fashion. The transmission time is proportional to the packets' length.

Determine the Kendall notation of the stochastic process in the following cases:

(a) The packets' length is exponentially distributed and the buffer of the node has an infinite capacity;

(b) The packets have a constant length and the buffer of the node can accommodate at most n packets;

(c) The packets have length L with probability p_L and length ℓ with probability p_ℓ, and there is no buffer to store the arrived packets.