Stochastic Models

A.A. 2019/2020

Assignement 1

Exercise 1. Consider the experiment consisting of picking two random marbles from a bag containing four marbles labelled with integers from 1 to 4.

(a) In the case when before picking the second marble, the first extracted is put back in the bag, calculate the sample space S_1 and its cardinality $|S_1|$;

(b) In the case without the replacement of the first ball, calculate the sample space S_2 and its cardinality $|S_2|$;

(c) In the case "without the replacement" of the first ball and by neglecting the extraction order, as it the lotto game, calculate the sample space S_3 and its cardinality $|S_3|$.

Exercise 2. Let us been given the following probabilities $\Pr(A) = 0.9$, $\Pr(B) = 0.8$ e $\Pr(A \cap B) = 0.75$. Calculate:

$$(a) \quad \Pr(A \cup B); \qquad (b) \quad \Pr(A \cap \overline{B}); \qquad (c) \quad \Pr(\overline{A} \cap \overline{B}).$$

Exercise 3. In a lot of 100 microchips, 20 of them are defective. Two microchips are randomly selected. Calculate:

(a) the probability that the first microchips was defective.

(b) the probability that the second microchips is defective given that the first was defective.

(c) the probability that both the extracted microchips were defective.

Exercise 4. In a multiple choice exam, there are 4 answers to a question. A student knows the right answer

(a) with probability 0.8;

(b) with probability 0.2;

(c) with probability 0.5.

If the student does not know the answer s/he always guesses with probability of success being 0.25. Given that the student marked the right answer, what is the probability s/he knows the answer.

Exercise 5. Let us consider an experiment consisting of n Bernoulli trials with parameter p. Calculate the probability that

(a) we get at least 1 success over n trial;

(b) we get k success over n trial;

(c) we get n success over n trial.

Exercise 6. The (normalized) temperature $X \in [0, 1]$ of an engine is a random variable whose probability density function is:

$$f_X(x) = \begin{cases} n \cdot (1-x)^{n-1} & \text{with} \quad 0 \leqslant x \leqslant 1,\ n > 0 \\ 0 & \text{otherwise} \end{cases}$$

Show that $f_X(x)$ is a valid probability density function. Then, independently of the previous result calculate its averaged value $\mathsf{E}(X)$.